The Influence of Perceived Constraints on Teachers' Problem-Solving Beliefs and Practices

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This paper describes data collected from a study that examined links between the use of problem-solving teaching approaches in primary mathematics classrooms and teachers' beliefs about the role of problem solving in learning mathematics. It appears that teachers held diverse views about the role of problem solving in mathematics teaching, that their reported practices were compatible with their beliefs, and that these beliefs and practices were influenced by identified, external constraints. The constraints included the grade level of the class, the school culture and time pressures.

Teachers generally report that they endorse the focus on problem solving in syllabus documents and agree that problem solving is an important life skill for students to develop (Anderson, 2000). Given the amount of policy advice and resource development, there are concerns about the limited opportunities for Australian students to engage in problems other than those of low procedural complexity (Stacey, 2003). This suggests that teachers' beliefs about the importance of problem solving are not being supported by actions in their classrooms.

There may be good reasons why problem solving seems to have a less prominent place in mathematics classrooms than may be intended. Frequently teachers' plans are thwarted by a range of contextual factors that include interruptions, and the urgent daily requirements that tend to take up so much of teachers' time. These factors are visible and easy to identify. Are there other factors that may be less visible and yet have a profound influence on teachers' actions? The study reported here explored primary school teachers' problem-solving beliefs and practices and sought to identify some of the less visible factors that may modify teachers' plans to implement problem-solving approaches in classrooms.

Background

Much of the research in the area of problem solving has focused on students, how they develop problem-solving abilities, and how teachers' can enhance these abilities (Schoenfeld, 1992). However, as well as developing students' problem-solving skills, it has been argued that when looking at teaching practice, the impact of teachers' beliefs is a critical factor (Thompson, 1992). In particular, teachers' professed beliefs are influenced by their actual beliefs, by their knowledge and interpretation of advice about teaching, by their use and understanding of curriculum materials, and by their own experiences as learners of mathematics (Putnam, 2003; Schoenfeld, 1999). Reported classroom practices appear to be influenced by reported beliefs, by actual practices in classrooms as well as by the constraints and opportunities that occur within the school context (Raymond, 1997). Few studies have examined the relationship between teachers' problem-solving beliefs and practices in detail.

Beliefs must be inferred and are therefore difficult to measure. Typically, data about beliefs are gathered using surveys, interviews or observations. Cooney, Shealy and Arvold (1998) noted that beliefs tend to be context specific, and they can be thought of as dispositions towards actions. Pajares (1992) suggested that beliefs are held with different intensities, and that beliefs influence perception in that they filter situations to make them more comprehensible. In recognising these important components of beliefs, Ambrose, Philipp, Chauvot and Clement (2003) developed a web-based instrument to explore seven specific beliefs about particular primary school contexts, as well as belief change with primary preservice students. The instrument used a range of open-ended questions rather than Likert scales so that the limitations of misinterpretation of items, lack of a context, and difficulty in determining the relative importance of an item, could be minimised. Hollingsworth (2003) also criticised the use of surveys to gather information about teaching practices as teachers can forget classroom events and may not even be aware of what they have done.

Recognising these limitations, the survey used in this study combined Likert scales and open-ended questions, and the data were complimented by interviews with a small sample of the respondents. To identify the extremes of teachers' beliefs and to facilitate categorisation of responses, an artificial continuum of teaching and learning was used. At one end of this continuum, mathematics is seen as a fixed body of facts to be delivered by teachers and internalised by students. Referred to as a *traditional* teaching approach, this perspective is associated with individual student work, rehearsal of routine questions, and reliance on textbooks or worksheets. This view may be accompanied by a belief that problem solving is an *end* and that problems should be presented to students after they have mastered basic facts and skills. At the other end of the continuum, termed a *contemporary* teaching approach, mathematics is seen as a dynamic subject to be explored and investigated. Classroom practices associated with this perspective usually involve group work and the use of non-routine questions that promote mathematical thinking, and the development of problem-solving skills. This teaching approach may be accompanied by a belief that problem solving is a *means* to learning mathematics. The concept of a continuum of teaching and learning with descriptions of particular perspectives was informed by the work of Levin and Ammon (1992) and Ernest (1991). Both described a range of approaches to teaching problem solving in mathematics classrooms.

Ernest (1991) proposed five ideologies of mathematics education and argued that there are three distinct views about the role of problem solving in the mathematics curriculum. Ernest argued that "Industrial Trainers" rejected problem solving as frivolous and a waste of time. He proposed that both the "Technological Pragmatists" and the "Old Humanists" viewed problem solving as additional content in the curriculum since they valued problems as important applications of mathematical content and processes and so problems were treated as objects of inquiry. Finally, Ernest suggested that the "Progressive Educators" and "Public Educators" held the third view of problem solving as a pedagogical approach and not an adjunct to the curriculum. As Ernest stated "the full incorporation of these processes into the curriculum, including problem posing, leads to a problem solving and investigational pedagogy" (p. 288).

It is relevant to seek to identify beliefs as well as other factors, or constraints, which may be moderating teachers' plans to implement problem solving in their classrooms. Most of the identified constraints from previous studies can be grouped into four broad categories: those relating to the teachers themselves (e.g., Jaworski, 1991), to students (e.g., Thompson, 1992), to school culture (e.g., Hoyles, 1992), and to system requirements (e.g., Clarke, 1993). This study aimed to identify the factors that interfered with the implementation of problem-solving practices in particular.

To explore what teachers believe and do in relation to problem solving, a study was designed to examine primary school teachers' beliefs about the role of problem solving in learning mathematics and their practices in classrooms (Anderson, 2000). Classroom observations were used to triangulate the data collected using the survey and interviews. In this paper, the data that aimed to discover the relationship between beliefs about mathematical problem solving and decision-making in teachers' classrooms, and the factors that teachers identified as constraining their implementation of problem-solving approaches is reported.

Methodology

The data reported here were some of the responses from 162 primary school teachers in New South Wales to a questionnaire based on similar instruments developed elsewhere (e.g., Raymond, 1997). To overcome the possibility of misinterpretation of survey items, examples were provided to illustrate the meaning of terms such as application, unfamiliar, and open-ended problems. To set the items in a meaningful context, as recommended by Ambrose et al. (2003), the first two questionnaire items sought information about teachers' problem-solving beliefs through the extent of their agreement with statements made by two imaginary teachers. Naomi's perspective, representing a traditional view, included statements like "students should learn algorithms before they do application and unfamiliar problems". Gwendolin's perspective represented a contemporary view. For example, she 'stated' that "students can learn most mathematical concepts by working out for themselves how to solve unfamiliar or open-ended problems" and "it is essential for students to explore their own ways of doing mathematics questions before being shown the teacher's methods". Respondents indicated whether they "strongly agreed", "agreed", "disagreed" or strongly disagreed" with the statements. It was anticipated that respondents' agreement or disagreement with the beliefs of each imaginary teacher would provide an indication of their reported beliefs.

Another item in the questionnaire listed twenty statements that sought data on teachers' reported practices. The items were chosen on the basis of strategies mentioned in the literature (e.g., Schoenfeld, 1992). For example: "you explain in detail what the students have to do to solve problems", and "you present unfamiliar and open-ended problems to the class with very little indication of how to solve them". Teachers rated the frequency of their use of each of the strategies as "hardly ever", "sometimes", "often", and "almost always". It was intended that the frequency with which teachers reported that they use these strategies would provide an indication of the perceived importance of the practices.

Data were also sought from interviews with nine teachers who represented the spread of problem-solving beliefs from the analysis of the questionnaires. This enabled confirmation of questionnaire analyses and provided an opportunity for further elaboration of teachers' intentions and identification of constraints that might appear to be operating in particular contexts.

Results and Discussion

The results presented here are inferred from two types of data: those from the questionnaire, and those from interviews with particular representative teachers. In this paper, particular items from two of the questions from the survey are discussed and relevant data from two of the teachers are presented.

Survey Data

In order to identify broad groups of responses, questionnaires were initially sorted into five categories, although for this paper, they have been grouped into three categories, referred to here as traditional, contemporary and mixed, based on each respondent's level of agreement with either the *traditional* or the *contemporary* perspective. Responses that were either "strongly agree" or "agree" were grouped together, as were those for "disagree" and "strongly disagree". A teacher was placed in the *traditional* category if there was agreement with five or more of the seven traditional statements as well as disagreement with five or more of the seven contemporary statements. The reverse was the case for those placed in the *contemporary* group. All other teachers were placed in the *mixed* group.

Of the 162 questionnaire responses, 23 (14%) were placed in the traditional category and 20 (12%) in the contemporary category. The following discussion uses the responses in these extreme categories. Responses to a sample of the traditional and contemporary belief statements by each of these groups of teachers are included in Table 1.

Table 1
Levels of Agreement (%) of the Traditional Teachers (n=23) and the Contemporary Teachers (n=20) to Selected Belief Statements

Belief statements	*Traditional* Agreement	*Contemporary* Agreement
students should learn algorithms before they do *application* and *unfamiliar* problems	100	10
application and *unfamiliar* problems are best left to the end of the topic in mathematics	61	5
mathematics lessons should focus on practising skills	78	15
mathematics lessons should focus on problems rather than on practice of algorithms	0	95
students can learn most mathematical concepts by working out for themselves how to solve *unfamiliar* and *open-ended* problems	0	95
it is essential for students to explore their own ways before being shown the teacher's methods	0	95

These responses indicate that there are teachers from the total population that appear to have polarised views, but within the groups there is strong agreement, matching the hypothesised dichotomy. It is interesting to compare the reported practices of these two groups. Table 2 presents the proportion of teachers in each of the two groups who reported using a particular strategy either "often" or "almost always".

Generally it seems that the reported beliefs and the reported practices are linked. The *traditional* teachers reported using strategies that are compatible with a transmissive style of teaching in that they frequently have students working alone, they prefer to provide detailed explanations, and most of this group frequently set exercises for skills practice. The *contemporary* teachers reported using practices that give responsibility to the students by encouraging group work, providing less initial explanation, encouraging individual recording, and allowing students to explore mathematical ideas.

Table 2

Proportions (%) of Each of the Traditional Teachers (N=23) and the Contemporary Teachers (N=20) Who Reported Using Selected Teaching Approaches "Often" or "Almost Always"

Selected teaching approaches	*Traditional*	*Contemporary*
you ensure that students work alone	43	0
you explain in detail what the students have to do to solve problems	61	15
you set exercises to allow the students to practise their skills	87	45
you model the problem solving process to the class	83	85
you discuss useful problem solving strategies (eg make a list, draw a diagram, work backwards)	83	85
you encourage the students to work in small, cooperative groups	43	80
you present unfamiliar and open-ended problems to the class with very little indication of how to solve them	0	35
you encourage students to record their own procedures and methods of solving problems	35	80
you pose open-ended problems to allow students to explore mathematical situations for themselves	9	55

Two items on the survey yielded similar responses for the two groups of teachers. Both the *traditional* and *contemporary* teachers reported frequently modelling problem-solving processes and discussing problem-solving strategies with their students. Perhaps the *difference* between the teachers is not so much on the value they attribute to problem solving but on how students learn to solve problems. Teachers' interpretations of these items could also explain this similarity in that the more traditional teachers could have interpreted the notion of 'modelling' as 'showing' students an algorithm they did not know that would enable them to solve a problem.

Of course, a critical issue is to identify some of the factors that influence the problem-solving beliefs and practices of teachers. One indication that practices may be influenced by constraints or external factors was determined from examining the teaching grade level of these groups of teachers. Table 3 indicates the proportion of each group teaching in each of the categories Years K-2, Years 3-4, Years 5-6 and those with a specialist role such as supporting students with learning difficulties.

Table 3

Proportions (%) of each of the traditional teachers (n=23) and contemporary teachers (n=20) in each of the teaching grade levels

	Years K-2	Years 3-4	Years 5-6	Specialist
Traditional	4	39	35	22
Contemporary	70	20	10	0

There seems to be some link between teaching grade level and association with *traditional* and *contemporary* views since 74% of the traditional teachers were teaching in Years 3 to 6 compared to 30% of the contemporary teachers. A possible explanation for support of the more traditional statements by teachers who were teaching in Years 3 to 6 is that algorithms are introduced and developed in these years for most students. As the curriculum mentions recall of facts, mental strategies, and written algorithms at this level, this might encourage teachers to focus on these skills and as a consequence, leave learning through problem solving until these have been established. This is an example of the way that perceived constraints influence the practices and perhaps even the beliefs of teachers.

Interview Data

To further explore the relationships between teachers' problem-solving beliefs and practices, some additional data from two teachers, representative of each of the *traditional* and *contemporary* groups are presented. Each of these teachers was interviewed to encourage elaboration of their questionnaire responses and to further explore the influence of contextual factors.

Lois (a pseudonym) had been teaching for 20 years and was teaching a Year 6 lower ability class. Overall, her questionnaire responses placed her in the *traditional* category. During the interviews, Lois' reported beliefs from the questionnaire were confirmed. Basic skills were viewed as necessary knowledge before students could do problem solving. In Lois' classroom, students were given short tasks with guidelines as to the process required for task completion. She believed that a structured approach was desirable in mathematics lessons. Lois stated

> it seems to me that students who aren't particularly capable in maths will only become amenable to sitting down … to solve a problem if they know some way of doing that.

Lois' decisions and planning seemed to centre on the needs of the students in her class. She reported that she seldom uses problem-solving approaches in her teaching since the students need practice on basic skills and generally find problem solving difficult. Interestingly, in response to questions about students' needs she added "if I had a top stream class or when I've had A classes then the answer would be different", thus emphasising that the constraints she felt from the class itself were a major determinant of her teaching approach. Lois similarly felt constrained by the textbook and assessment regimes used in the school, and the demands from parents for preparation for competitive examinations.

Lois viewed problem solving as an added extra to the curriculum and as an object of inquiry, particularly suitable for more able students as an extension activity. Thus contrasting with the view that problem solving can be a process of inquiry, or a teaching approach suitable for all students. In fact, Lois rejected this view and was highly critical of this approach during the interview.

Mary (a pseudonym) had been teaching for four years and her responses placed her in the *contemporary* category. At the time of questionnaire completion, Mary was teaching a Year 2 class in a school with a large proportion of less experienced teachers. At the time of the interviews she was teaching a mixed ability Year 4 class in a different school with an experienced staff who used more traditional approaches to teaching mathematics.

On her questionnaire, Mary indicated that she often used open-ended and application problems, sometimes used unfamiliar problems, and rarely presented exercises to her students. Her response to why she preferred to use such problems was that

> Open-ended problems allow children to bring their own knowledge and strategies to the task as well as respond at their own level. My children write their own problems which are often in the form of application problems.

During the interviews, Mary indicated that she would now probably qualify some of the statements she had rejected or supported in the questionnaire. While she still agreed that “problem solving can actually teach you some of the basic number facts”, she now felt that this was not the case for all students. To support her change in views she said “I think I was going too far into problem solving and not giving them enough of the traditional stuff”. However, she still supported the use of problem-solving approaches since she stated

> I like the idea of using problem solving because a lot of the problems you use relate maths to the real world and I know that every time I mention mathematics in the real world with my class they immediately attend much better than they normally would ... it does motivate them because they like a challenge. Any child likes a challenge.

For Mary, there appeared to be some moderation of her views from the questionnaire to the interview that seemed to be influenced by the new school setting with its different culture as well as a change from teaching in the lower primary to middle primary school years. This latter factor was acknowledged when Mary stated, “the older the child the more formal you tend to become”. She agreed that there was more of a place for algorithm practice in Years 3 to 6 and that this meant teachers were more inclined to set exercises in these grades than in the lower primary grades.

In her second school, Mary encountered a more conservative teaching staff whose influence seemed to cause her to reflect on her practice and to question her views about mathematics teaching and learning. It appeared that she was still convinced that there was a place for problem-solving approaches but she now considered the need to present students with more skills practice and a more structured learning approach to algorithms. Mary voiced some discomfort in expressing this change in views but she had clearly considered both perspectives and was prepared to discuss reasons for her change in views. She confided that another factor that was impacting on her current approach was the lack of time available for mathematics lessons each week.

Mary believed in problem solving as a process of inquiry. It appeared that her initial enthusiasm in using problem-solving approaches in the first school was influenced by her experiences in preservice education courses. She had embraced much of the advice from the problem-solving literature. In practice, Mary had found it easier to implement such approaches in the Kindergarten to Year 2 classes than in her Year 4 class. In addition, her first school had a younger, less-experienced staff, possibly with a similar enthusiasm for such ideas. As with Lois, Mary’s views about approaches to teaching mathematics were influenced by the year level she was teaching as well as other factors associated with the culture of the school and the time pressures she was experiencing.

Conclusion

The data support the existence of a dichotomy in relation to primary school teachers’ beliefs about the role of problem solving in learning mathematics. In responding to a survey, there were some teachers who reported holding *traditional* views that were quite distinct from another group who reported support for *contemporary* views. These beliefs translated into different reported classroom practices. The interview data suggest that these differences are related to particular constraints influencing beliefs and intentions. One of these relates to the grade level of the class, another to the school culture, and another to time pressures. Hoyles (1992) refers to this as “beliefs in practice” suggesting that teachers’ beliefs and practices are not merely a reflection of individual preferences, but are also determined by the particular school context.

Hence, a general belief that mathematics becomes more formal in the higher grades of primary school and overall beliefs about how ‘formal’ mathematics should be taught are major constraints to teachers’ implementation of problem-solving approaches. Clearly, if problem solving is viewed as an important factor in mathematics learning and teaching, education policy, curriculum, and teacher development initiatives must acknowledge the constraints on teachers. Teacher professional development needs to include opportunities for "teachers to wonder, to doubt, to consider what might be, to reflect, and most important, to be adaptive" (Cooney et al., 1998, p. 332). Any proposals for change should incorporate proposals for more problem-solving approaches for teaching older primary school grades, recommendations about ways to assist changing school cultures, and a review of the expectations on teachers that create little time for reflection on practice.

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